OBITUARY NOTICES OF FELLOWS DECEASED.

ARCHIBALD SMITH, only son of James Smith, of Jordanhill, Renfrewshire, was born on the 10th of August, 1813, at Greenhead, Glasgow, in the house where his mother's father lived. His father, who also was a Fellow of the Royal Society, had literary and scientific tastes with a strongly practical turn, fostered no doubt by his education in the University of Glasgow and his family connexion with some of the chief founders of the great commercial community which has grown up by its side. In published works on various subjects he left enduring monuments of a long life of actively employed leisure. His discovery of different species of Arctic shells, in the course of several years' dredging from his yacht, and his inference of a previously existing colder climate in the part of the world now occupied by the British Islands, constituted a remarkable and important advancement of Geological Science. In his 'Voyage and Shipwreck of St. Paul,' a masterly application of the principles of practical seamanship renders St. Luke's narrative more thoroughly intelligible to us now than it can have been to contemporary readers not aided by nautical knowledge. Later he published a 'Dissertation on the Origin and Connexion of the Gospels;' and he was engaged in the collection of further materials for the elucidation of the same subject up to the time of his death, at the age of eighty-five. Archibald Smith's mother was also of a family distinguished for intellectual activity. Her paternal grandfather was Dr. Andrew Wilson, Professor of Astronomy in the University of Glasgow, whose speculations on the constitution of the sun are now generally accepted, especially since the discovery of spectrum-analysis and its application to solar physics. Her uncle, Dr. Patrick Wilson, who succeeded to his father's Chair in the University, was author of papers in the 'Philosophical Transactions' on Meteorology and on Aberration.

Archibald Smith's earliest years were chiefly passed in the old castle of Roseneath. In 1818 and 1819 he was taken by his father and mother to travel on the continent of Europe. Much of his early education was given him by his father, who read Virgil with him when he was about nine years old. He also had lessons from the Roseneath parish schoolmaster, Mr. Dodds, who was very proud of his young pupil. In Edin-

burgh during the winters 1820–22 he went to a day-school; and after that, living at home at Jordanhill, he attended the Grammar School of Glasgow for three years. As a boy he was extremely active, and fond of every thing that demanded skill, strength, and daring. At Roseneath he was constantly in boats; and his favourite reading was any thing about the sea, commencing no doubt with details of adventurers and buccaneers, but going on to narratives of voyages of discovery, and to the best text-books of seamanship and navigation as he grew older. He had of course the ordinary ardent desire to become a sailor, incidental to boys of this island; but with him the passion remained through life, and largely influenced the scientific work by which he has conferred never-to-be-forgotten benefits on the marine service of the world, and made contributions to nautical science which have earned credit for England among maritime nations. He was early initiated into practical seamanship under his father's instruction, in yacht-sailing. He became an expert and bold pilot, exploring and marking passages and anchorages for himself among the intricate channels and rocks of the West Highlands, when charts did not supply the requisite information. His most loved recreation from the labours of Lincoln's Inn was always a cruise in the West Highlands. In the last summer of his life, after a naturally strong constitution had broken down under the stress of mathematical work on ships' magnetism by night, following days of hard work in his legal profession, he regained something of health and strength in sailing about with his boys in his yacht, between the beautiful coasts of the Frith of Clyde, but not enough, alas, to carry him through unfavourable influences in the winter that followed.

In 1826 he went to a school at Redland, near Bristol, for two years; and in 1828 he entered the University of Glasgow, where he not only began to show his remarkable capacity for mathematical science in the classes of Mathematics and Natural Philosophy, but also distinguished himself highly in Classics and Logic. Among his fellow students were Norman Macleod and Archibald Campbell Tait, with both of whom he retained a friendship throughout life. After completing his fourth session in Glasgow, he joined in the summer of 1832 a reading party, under Hopkins, at Barmouth in North Wales, and in the October following commenced residence in Trinity College, Cambridge.

While still an undergraduate he wrote and communicated to the Cambridge Philosophical Society a paper on Fresnel's wave-surface. The mathematical tact and power for which he afterwards became celebrated were shown to a remarkable degree in this his first published work. Fresnel, the discoverer of the theory, had determined analytically the principal sections of the wave-surface, and then guessed its algebraic equation. This he had verified, by calculating from it the perpendicular from the centre to the tangent plane; but the demonstration thus ob-

tained was so long that he suppressed it in his published paper. Ampère by sheer labour had worked out a direct analytical demonstration, and published it in the 'Annales de Chimie et de Physique'*, where it occupies thirty-two pages, and presents so repulsive an aspect that few mathematicians would be pleased to face the task of going through it. With these antecedents, Archibald Smith's investigation, bringing out the desired result directly from Fresnel's postulates by a few short lines of beautifully symmetrical algebraic geometry, constitutes no small contribution to the elementary mathematics of the undulatory theory of light. It was one of the first applications in England, and it remains to this day a model example, of the symmetrical method of treating analytical geometry, which soon after (chiefly through the influence of the 'Cambridge Mathematical Journal') grew up in Cambridge, and prevailed over the unsymmetrical and frequently cumbrous methods previously in use.

In 1836 he took his degree as Senior Wrangler and first Smith's Prizeman, and in the same year he was elected to a Fellowship in Trinity College.

Shortly after taking his degree, he proposed to his friend Duncan Farquharson Gregory, of the celebrated Edinburgh mathematical family, then an undergraduate of Trinity College, the establishment of an English periodical for the publication of short papers on mathematical subjects. Gregory answered in a letter of date December 4th, 1836, cordially entering into the scheme, and undertaking the office of editor. Being, however, on the eve of the Senate-House examination for his degree, he adds, "But all this must be done after the degree; for 'business before pleasure,' as Richard said when he went to kill the king before he murdered the babes." The result was, the 'Cambridge Mathematical Journal,' of which the first number appeared in November 1837. It was carried on in numbers, appearing three times a year under the editorship of Gregory, until his death, and has been continued under various editors, and with several changes of name, till the present time, when it is represented by the 'Quarterly Journal of Mathematics' and the 'Messenger of Mathematics.' The original 'Cambridge Mathematical Journal' of Smith and Gregory, containing as it did many admirable papers by Smith and Gregory themselves, and by other able contributors early attracted to it, among whom were Greatheed, Donkin, Walton, Sylvester, Ellis, Cayley, Boole, inaugurated a most fruitful revival of mathematics in England, of which Herschel, Peacock, Babbage, and Green had been the prophets and precursors.

It is much to be regretted that neither Cambridge, nor the university of his native city, could offer a position to Smith, enabling him to make the mathematical and physical science, for which he felt so strong an inclination, and for which he had so great capacity, the professional

* Volume for 1828.

work of his life. Two years after taking his degree he commenced reading law in London; but his inclination was still for science. Relinquishing reluctantly a Trinity Lectureship offered to him by Whewell in 1838, and offered again and almost accepted in 1840, resisting a strong temptation to accompany Sir James Ross to the Antarctic regions on the scientific exploring expedition of the 'Erebus' and 'Terror' in 1840–41, and regretfully giving up the idea of a Scottish professorship, which, during his early years of residence in Lincoln's Inn, had many attractions for him, he finally made the bar his profession. But during all the long years of hard work through which he gradually attained to an important and extensive practice, and to a high reputation as a Chancery barrister, he never lost his interest in science, nor ceased to be actively engaged in scientific pursuits; and he always showed a lively and generous sympathy with others, to whom circumstances (considered in this respect enviable by him) had allotted a scientific profession.

About the year 1841 his attention was drawn to the problem of ships' magnetism by his friend Major Sabine, who was at that time occupied with the reduction of his own early magnetic observations made at sea on board the ships 'Isabella' and 'Alexander' on the Arctic Expedition of 1818, and of corresponding magnetic observations which had been then recently made on board the 'Erebus' and 'Terror' in Capt. Ross's Antarctic Expedition of 1840–41. The systematic character of the deviations, unprecedented in amount, experienced by the 'Isabella' and 'Alexander' in the course of their Arctic voyage, had attracted the attention of Poisson, who published in 1824, in the 'Memoirs of the French Institute,' three papers containing a mathematical theory of magnetic induction, with application to ships' magnetism. The subsequent magnetic survey of the Antarctic regions, of which by far the greater part had to be executed by daily observations of terrestrial magnetism on ship-board, brought into permanent view the importance of Poisson's general theory; but at the same time demonstrated the necessity for replacing his practical formulæ by others, not limited by certain restrictions as to symmetry of the ship, which he had assumed for the sake of simplicity. This was the chief problem first put before Smith by Sabine; and his solution of it was the first great service which he rendered to the practical correction of the disturbance of the compass caused by the magnetism of ships. Twenty years later the work thus commenced was referred to in the following terms by Sir Edward Sabine*, in presenting, as President of the Royal Society, the Royal Medal which had been awarded to Archibald Smith for his investigations and discoveries in ships' magnetism:— * * * "Himself "a mathematician of the first order, and possessing a remarkable facility "(which is far from common) of so adapting truths of an abstract cha-

* Proceedings of the Royal Society, Nov. 30, 1865, vol. xiv. p. 499.

" racter as to render them available to less highly trained intellects, he
" derived at my request, from Poisson's fundamental equations, simple
" and practical formulæ, including the effects both of induced magnetism
" and of the more persistent magnetism produced in iron which has
" been hardened in any of the processes through which it has passed.
" The formulæ supplied the means of a sufficiently exact calculation
" when the results were finally brought together and coordinated. They
" were subsequently printed in the form of memoranda in the account
" of the survey in the 'Philosophical Transactions' for 1843, 1844, and
" 1846.

" The assistance which, from motives of private friendship and scien-
" tific interest, Mr. Smith had rendered to myself, was from like motives
" continued to the two able officers who had successively occupied the
" post of Superintendent of the Compass Department of the Navy; and
" the formulæ for correcting the deviation, which he had furnished to
" me, reduced to simple tabular forms, were published by the Admiralty
" in successive editions for the use of the Royal Navy.

" As, in the course of time, the use of steam machinery, the weight of
" the armament of ships of war, and generally the use of iron in vessels,
" increased more and more; the great and increasing inconvenience
" arising from compass irregularities were more and more strongly felt,
" and pressed themselves on the attention of the Admiralty and of
" naval officers.

" An entire revision of the Admiralty instructions became necessary;
" Mr. Smith's assistance was again freely given; and the result was the
" publication of the 'Admiralty Manual' for ascertaining and applying
" the deviations of the compass caused by the iron in a ship.

" The mathematical part of this work, which is due to Mr. Smith,
" seems to exhaust the subject, and to reduce the processes by simple
" formulæ and tabular and graphic methods, to the greatest simplicity of
" which they are susceptible. Mr. Smith also joined with his fellow-
" labourer, Captain Evans, F.R.S., the present Superintendent of the
" Compass Department of the Navy, in laying before the Society several
" valuable papers containing the results of the mathematical theory
" applied to observations made on board the iron-built and iron-plated
" ships of the Royal Navy."

This is not an occasion for explaining in detail the elaborate investigations sketched in the preceding statement by Sir Edward Sabine; but the writer of the present notice, having enjoyed the friendship of Archibald Smith since the year 1841, and having had many opportunities, both in personal intercourse and by letters, of following the progress through thirty years of his work on ships' magnetism, may be permitted a brief reference to some of the points which have struck him as most remarkable :—

1. Harmonic reduction of observations.
2. Practical expression of the full mathematical theory.
3. Heeling error.
4. Dygograms.
5. Rule for positions of needles on compass card, with dynamical and magnetic reasons.

1. *Harmonic reduction of observations.*—The disturbance of the compass produced by the magnetism of a ship is found by observation to be the same, to a very close degree of approximation, when the ship's head is again and again brought to the same bearing, no great interval of time having intervened, and no extraordinary disturbance by heavy sea or otherwise having been experienced in the interval. Overlooking these restrictions for the present, we may therefore say, in Fourier's language, that the disturbance of the compass is a periodic function of the angle between the vertical plane of any line fixed relatively to the ship, and any fixed vertical plane, when the ship, on "even keel" or with any constant inclination, is turned into different azimuths—the period of this function being four right angles. Hence also the disturbance of the compass is a periodic function of the angle between the vertical plane of the chosen line moving with the ship, and the vertical plane through the magnetic axis of the compass. The line moving with the ship being taken as a longitudinal line drawn horizontally from the stern towards the bow, and the fixed vertical plane being taken as the magnetic meridian, the angle first mentioned is called for brevity "the ship's *magnetic course*," and the other "the ship's *compass course*."

One of Smith's earliest contributions to the compass problem was the application of Fourier's grand and fertile theory of the expansion of a periodic function in series of sines and cosines of the argument and its multiples, now commonly called the harmonic analysis of a periodic function. To facilitate the practical working out of this analysis, he gave tables of the products of the multiplication of the sines of the "rhumbs" by numbers, and by arcs in degrees and minutes; also tabular forms and simple practical rules for performing the requisite arithmetical operations. These tables, tabular forms, and rules, just as Smith gave them about thirty years ago, are in use in the Compass Department of the Admiralty up to the present time. From every ship in Her Majesty's Navy, in whatever part of the world, a table of observed deviations of the compass, at least once a year is sent to the Admiralty, and is there subjected to the harmonic analysis. The observations having been accurately and faithfully made, a full history of the magnetic condition of the ship is thus obtained, and want of accuracy, or want of faithfulness, if there has been any, is surely detected. The rigorous carrying out of this system, with all the method and business-like regularity characteristic of the scientific departments of our Admiralty, has undoubtedly done

more than any thing else to promote the usefulness of the compass, and to render its use safe throughout the British Navy. Smith's tables and forms for harmonic analysis have proved exceedingly valuable in many other departments of practical physics besides ships' magnetism. The writer of this article found them most useful fifteen years ago in reducing for the Royal Society of Edinburgh Forbes's observations of the underground temperature of Calton Hill, the Experimental Gardens, and Craigleith Quarry, in the neighbourhood of Edinburgh; and the forms, with a suitable modification of the tables, have proved equally useful in the harmonic analysis of tidal observations for various parts of the world, carried out by the Tidal Committee of the British Association, with the assistance of sums of money granted in successive years from 1868 to 1872.

2. *Practical expression of the full mathematical theory.*—Poisson himself, in making practical application of his theory, had simplified it by assuming particular conditions as to symmetry of the iron in the ship, and even with these restrictions had left it in a form which seemed to require further simplification before it could be rendered available for general use. Airy, in taking up the problem with this object, at the request of the Admiralty in the year 1839, founded his calculations on a supposition that, " by the action of terrestrial magnetism every particle " of iron is converted into a magnet whose direction is parallel to that of " the dipping needle, and whose intensity is proportional to the intensity " of terrestrial magnetism." This supposition, which is approximately true only for the ideal case of the iron of the ship being all in the shape of globes placed at such considerable distances from one another as not to exercise mutual influence to any sensible degree, leads to a law of dependence between the ship's force on the compass needle, and the angular coordinates of the ship, which differs from that of the complete theory, as shown afterwards by Smith, only in the want of his constant term A of the harmonic development,—a difference which, in ordinary cases, does not vitiate sensibly the practical application. In introducing the supposition, Airy correctly anticipated that it would in general lead to results sufficiently accurate and complete for practical purposes. But he said " it would have been desirable to make the calculations on " Poisson's theory, which undoubtedly possesses greater claims on our " attention (as a theory representing accurately the facts of some very " peculiar cases) than any other. The difficulties, however, in the appli- " cation of this theory to complicated cases are great, perhaps insuper- " able." These difficulties were wholly overcome by the happy mathematical tact of Archibald Smith, who reduced the full expression of Poisson's theory, including the effect of permanent magnetism, the great practical importance of which had been discovered by Airy, to a few simple and easily applied formulæ. [See Appendix to this notice.] These

formulæ are now in regular use in the Compass Department of the Admiralty, for the practical deduction of rigorous results from the harmonic analysis already referred to. In fact the full expression of the unrestricted theory, as given by Archibald Smith in Part III. of the 'Admiralty Manual,' is even simpler and more ready for ordinary use than the partial and restricted expressions which Poisson and Airy had given for practical application of the theory.

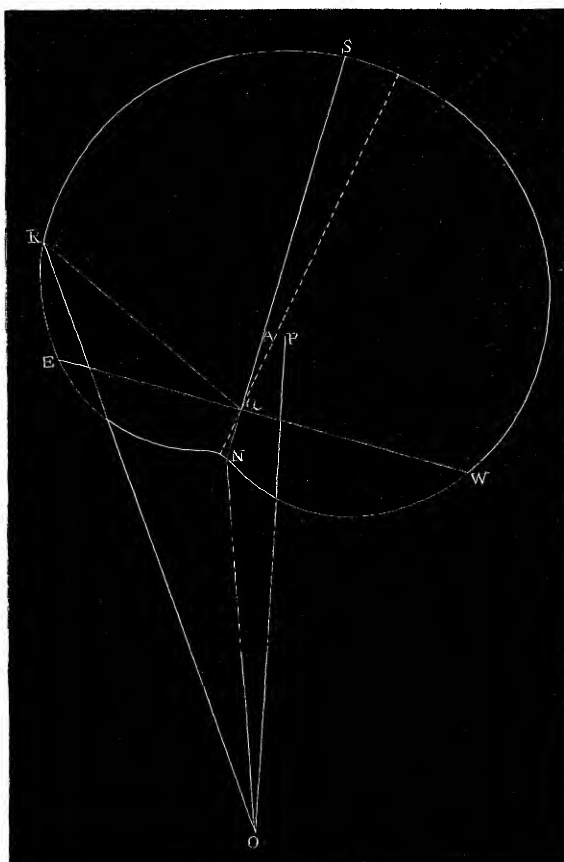
3. *Heeling error.*—Poisson's general formulæ express three rectangular components of the resultant force at the point where the compass is placed, due to the magnetism induced in the ship by the terrestrial magnetic force. To these Airy added the components of force due to permanent magnetism of the ship's iron, which, though not ignored by Poisson, had been omitted by him, because, considering the probability of scattered directions of the magnetic axes of permanent magnetism in the isolated masses of iron existing in wooden ships and their armaments, he justly judged that permanent magnetism could not seriously disturb a properly placed compass in a wooden ship ; and iron ships were scarcely contemplated in those days. This general theory of Poisson and Airy expresses the resultant force in terms of three angular coordinates, specifying the position of the ship. In the practical application these coordinates are most conveniently taken as :—(1) the ship's "magnetic course," defined above ; (2) the inclination of the longitudinal axis of the ship to the horizon ; (3) the inclination to the horizon of a plane drawn through this line perpendicular to the deck. The second coordinate has no name and is of no importance in the compass problem ; for under steam, or even under sail, the average inclination of the longitudinal axis (chosen as horizontal for the ship in still water) is never so great as to produce any sensible effect on the compass disturbance, and the magnetic effects of pitching in the heaviest sea are not probably ever so great as to produce any seriously inconvenient degrees of oscillation in the compass card. The third coordinate is called the "heel ;" and its magnetic effect on the compass is called "the heeling error." The heeling error was investigated by Airy in his earliest work on the compass disturbance ; but at that time, when iron sailing ships were comparatively rare, he confined his ordinary practical correction of compass error to the case of a ship in different azimuths on even keel. Since that time the heeling error has come to be of very serious practical importance, on account of the great number of iron sailing ships, and of screw steamers admitting of being pressed by sail to very considerable degrees of "heel." Archibald Smith took up the question with characteristic mathematical tact and practical ability, and gave the method for correcting the heeling error—which is now, I believe, universally adopted in the Navy, and too frequently omitted (without the substitution of any other method) in the mercantile marine.

4. *Dygograms.*—This is the name given by Smith to diagrams exhibiting the magnitude and direction of the resultant of the terrestrial magnetic force and the force of the ship's magnetism at the point occupied by the compass. The solution of the problem of finding for a ship in all azimuths on even keel the dygogram of the whole resultant force is given by him in the chapter headed "Ellipse and Circle," of the 'Admiralty Manual,' Appendix 2 (3rd edition, 1869, page 169–171). But it is only for horizontal components of force that he has put dygograms into a practical form; and for this case, which includes the whole compass problem of ordinary navigation, his dygograms are admirable both for their beauty and for their utility. "Dygogram Number I." is the curve locus of the extremity of a line drawn from a fixed point, O, in the direction, and to a length numerically equal to the magnitude, of the horizontal component of the resultant force experienced by the needle when the ship is turned through all azimuths. This curve (however great the deviations of the compass) he proves to be the Limaçon of Pascal—that is to say, the curve (belonging to the family of epitrochoids) described by the end of an arm rotating in a plane round a point, which itself is carried with half angular velocity round the circumference of a fixed circle in the same plane. The length of the first-mentioned arm is equal to the maximum amount of what is called (after Airy) the quadrantal deviation; the radius of the circle last mentioned is the maximum amount of what Airy called the polar magnet deviation, and Smith the semicircular deviation. (When, as the writer of this article trusts before long will be universally the case*, the quadrantal deviation is perfectly corrected by Airy's method of soft iron correctors, the dygogram Number I. will be reduced to a circle.) Besides the form of the curve in any particular case, which depends on the ratio of the first-mentioned radius to the second, to complete the diagram and use it we must know the position of the fixed point through which the resultant radius-vector is to be drawn, and must show in the diagram the magnetic bearing of the ship's head, for which any particular point of the curve gives the resultant force. Smith gave all these elements by simple and easily executed constructions, in the first and second editions of the 'Admiralty Manual.' In the third edition he substituted, for his first method of construction of the dygogram curve, a modification of it due to Lieut. Colongue of the Russian Imperial Navy and of the Imperial Compass Observatory, Cronstadt, and added several elegant constructions, also due to Lieut. Colongue, for the geometrical solution of various compass problems, by aid of the dygogram Number I.

* The barrier against this being done hitherto has been the perniciously great length of the compass needles used at sea, the shortest being about six inches. For a standard compass the needles ought not to be more than half an inch long.

The annexed diagram is the dygogram Number I. for the 'Warrior,' drawn accurately (by aid of a circular board rolling upon a fixed circular board of equal diameter, in the manner described by Smith in the 'Admiralty Manual,' Appendix II., under the heading "Mechanical Construction of Dygogram No. I."), according to the following data deduced from observations made at Spithead in October 1861. The notation $\mathfrak{A}$, $\mathfrak{B}$,

$\lambda H = 1.$
$\mathfrak{A} = -0{\cdot}017.$
$\mathfrak{B} = -0{\cdot}408.$
$\mathfrak{C} = -0{\cdot}090.$
$\mathfrak{D} = +0{\cdot}156.$
$\mathfrak{E} = +0{\cdot}015.$



Rule for using Dygogram No. I.—In the diagram Q is a fixed point of the "Limaçon," called "the pole of the dygogram." It lies in the axis of symmetry, which is indicated by a dotted line. NQS, EQW are two lines through Q at right angles to one another; and O, P are two points, in positions fixed by the ship's magnetic elements. The length OP represents "mean force on compass to north" (λH). Take any point R on the curve, such that NOR is equal to the ship's "magnetic course;" then is ROP the "deviation" of the compass, and OR represents the horizontal component of the force on it.

$\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, is that which was introduced by Smith when he first substituted the rigorous formulæ for the approximate harmonic formulæ which had previously sufficed: it is explained in the Appendix to this notice.

Dygogram Number II. may be deduced from dygogram Number I. by attaching a piece of paper to the half-speed revolving arm, and letting the tracing-point of the limaçon leave its trace also on this paper, which will be a circle, while at the same time the fixed point from which the resultant radius-vector is drawn will trace another circle on the moving paper. The fresh diagram thus obtained consists of two circles. Mark one of these circles (the second in the order of the preceding description) with the points of the compass*, like a compass card; or (better) mark simply degrees all round from North taken as zero; and mark with degrees counted in reverse direction the other circle, which, for brevity, will be called the auxiliary circle. Mark the ship's compass course on the circumference of the ideal compass card. From this point to the corresponding point on the auxiliary circle draw a straight line. The direction of this line shows by the parallel to it, through the centre of the ideal compass card, the compass course corresponding to any chosen magnetic course. The length of the line, drawn in the manner described, represents the horizontal resultant force of the earth and ship, at the point occupied by the compass needle, in terms of the radius of the ideal compass card, as unity. The writer of the present article believes that this construction will yet prove of very great practical utility, although hitherto it has not come into general use†. Its geometrical beauty attracted the notice even of Cayley, who has contributed to the Admiralty Compass Manual a second method of solving, by means of it, one of Smith's compass problems.

Construction from ship's and earth's magnetic elements. With O as centre and OH equal to "mean force on compass to north" (λH) describe a circle. Make

$$NL=\mathfrak{A},\ OB=-\mathfrak{B};\ BC=-\mathfrak{C};\ CD=-\mathfrak{D};\ Dh=-\mathfrak{E};$$

with C as centre describe a circle through h.

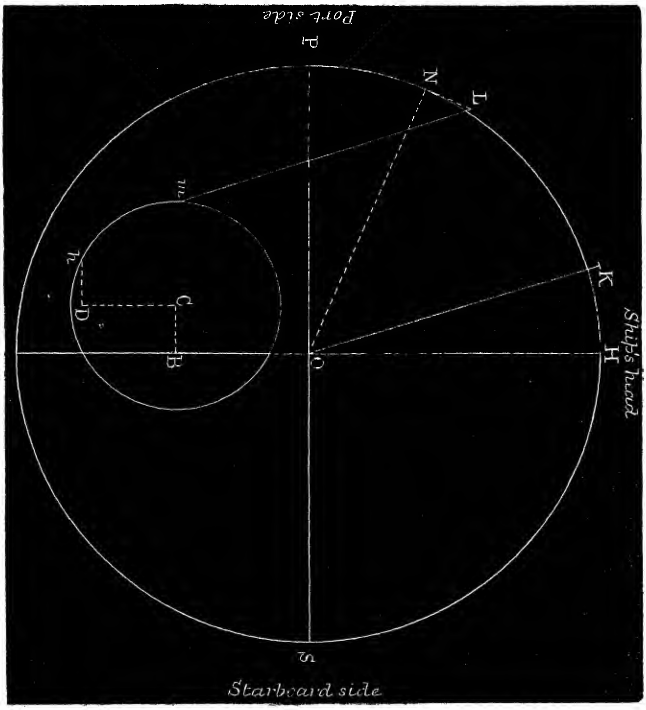
The following diagram shows (for an ideal case, as possibly a turret ship of the future, with very large values of the usually small magnetic elements $\mathfrak{A}$ and $\mathfrak{E}$) the Dygogram of two circles, modified to suit the Chinese compass (or needle unloaded with compass card, which is undoubtedly the compass of the future). This modification is also conve-

* The ancient system of marking 32 points on the compass card, and specifying courses in terms of them, has always been very inconvenient, and is now beginning to be generally perceived to be so.

† A short demonstration of it, deduced directly from Smith's fundamental formulæ, is appended to the present article for the sake of mathematical readers who may not have the Admiralty Compass Manual at hand.

nient for the theoretical explanation and proof appended to the present notice.

Dygogram No. II.



Use.—Make $\overset{\frown}{hm}$ equal in angular value to $\overset{\frown}{NH}$; then OK, parallel and equal to *m*L, shows direction of needle and magnitude of horizontal component force on it when correct magnetic north is in direction ON, and ship's head OH: NOH being ship's "magnetic course," KOH is the corresponding "compass course."

5. *Rule for positions of needles on compass card, with dynamical and magnetic reasons.*—In 1837 a Committee, consisting of Captain Beaufort, Hydrographer to the Admiralty, Captain Sir J. C. Ross, R.N., Captain Johnson, R.N., Major Sabine, R.A., and Mr. S. H. Christie, was appointed to remedy defects of the compasses at that time in use in Her Majesty's fleet, and to organize a system of compass management ashore and afloat. The labours of that Committee have conferred signal benefit, not only on the British Navy, but on the navies and mercantile marine services of all nations—in the 'Admiralty Standard Compass,' and in the establishment in 1843 of the British Admiralty Compass Department. The qualities of the magnetic needles and their arrangement on the card occupied much

attention of the Committee. Smith's attention was called to the subject by his friend Sabine; and he gave a rule for placing the needles, which was adopted by the Committee, and has ever since been followed in the construction of the Admiralty compass. The rule is, that when there are two needles used they should be placed with their ends on the compass card at 60° on each side of the ends of a diameter; and that when (as in the Admiralty Standard Compass) there are four needles, they should be placed with their ends at 15° and 45° from the ends of the diameter. The object of this rule was to give equal moments of inertia round all horizontal axes, and so to remedy the "wabbling" motion of the compass card when balanced on its pivot, which has been found inconvenient. Captain Evans, in a letter recently received from him by the writer of this notice, says that the "wabbling" motion has been satisfactorily corrected by this arrangement of needles; "it is transformed into a '*swimming*' motion."

About twenty years later it was discovered that the same arrangement gives, by a happy coincidence, a very important magnetic merit to the Admiralty compass, which had not been contemplated by Smith when he first gave his rule. To explain this, it must be premised that practical compass-adjusters had experienced difficulties in correcting the compass deviation of certain ships by Airy's method (which consists in using soft iron to correct the quadrantal deviation, and permanent magnets to correct the semicircular), and had reported that in such cases they had found it advantageous to substitute compasses with two needles for a single-needle compass. The attention of Captain Evans was drawn to this subject by the observations made in the 'Great Eastern' on her experimental voyage from the Thames to Portland, and afterwards when she was lying at Holyhead and Southampton, from which he found that although the deviations had been carefully corrected by Mr. Gray, of Liverpool, with magnets and soft iron, and were in fact nearly correct on the cardinal and quadrantal points, there were errors of between 5° and 6° on some of the intermediate points. These observations indicated the existence of a considerable error, which was neither semicircular nor quadrantal, and thus apparently some source of error which had not been taken into account by Airy in his plan for correction. To explain the cause of these and similar results in other ships, previously considered to be anomalous, Captain Evans instituted a series of experiments with compasses, and magnets and soft iron placed in different positions with respect to them. He soon found that the greatness of the supposed anomaly in the 'Great Eastern' depended on the unusually great length of the needles of her standard compass (two needles* of $11\frac{1}{2}$ inches in

* Compass needles becoming larger with the ships, by a process of "Artificial Selection" unguided by intelligence, have sometimes attained to the monstrous length of 15 inches, or even more, in some of the great modern passenger-steamers fitted out by owners regardless of expense, and only desiring efficiency, trusting to instrument-makers

length, placed near each other on the card). The results of the observations and experiments, reduced by aid of Napier's graphic method, and subjected to a thorough harmonic analysis, are described in a joint paper by Smith and Evans, published in the Transactions of the Royal Society for 1861. They show, in the expression for the deviation, sextantal and octantal terms* very large in the case of the 'Great Eastern,' and comparatively small when the Admiralty standard compass was tested in circumstances otherwise similar. Whether single needles or double needles were used, it was found that the smaller the needle the smaller were the sextantal and octantal terms. Single needles gave greater terms of this class than double needles of the same magnitude, arranged as in the Admiralty compass.

The merit of giving almost evanescent sextantal and octantal terms,

of the highest name. Reversion to the old Chinese species, with single needle less than an inch long and unloaded by a compass card, would be an improvement on the present ordinary usage of first-class ocean steamers.

The direction of part of the reactionary improvement required is clearly pointed out in the following note on the comparative merits of large and small compasses, extracted from Captain Evans's 'Elementary Manual for the Deviation of the Compass in Iron Ships:'—
"Of late years much diversity in practice has prevailed as to the size of compasses for "use on board ship. The Admiralty Standard card of $7\frac{1}{2}$ inches diameter, for example, "is fitted with needles the maximum lengths of which are $7\frac{1}{3}$ inches, while in large "passenger steam-vessels the needles are frequently 12 to 15 inches, and even longer. "The chief object in the employment of large compasses is to enable the helmsmen "to steer to degrees; and a more accurate course is thus presumed to be preserved."

"With reference to this increased size, it must be observed that competent authorities "limit the length of efficient compass needles to 5 or 6 inches; beyond this limit an "increase of length is alone accompanied by an increase of directive power in the "same proportion; and if the thickness of the needle be preserved, the weight, and "consequently the friction, increase in the same ratio. No advantage of directive "power is therefore gained by increase in length; but with the increased weight of the "card and appendages the increase of friction probably far exceeds the increase of direc- "tive force: sluggishness is the result, which is further exaggerated by the extreme slow- "ness of oscillation of long needles compared with short needles."

"Large cards, however convenient in practice, are therefore not without danger; for "the course steered may deceive the seamen by seeming right to the fraction of a "degree, but which avails little if the card is wrong half a point, and the ship in con- "sequence hazarded. In the opinion of the writer the present Admiralty standard "card is as large as should be used for the purposes of *navigation*, and that, as regards "safety in the long, steady, and fast ship, the choice is really between the Admiralty "card and a smaller one. In short the case may be thus stated: the smaller a card "the more correctly it points; the larger a card the more accurately it can be read."

When the needles of a standard compass are reduced to something like half an inch in length, and not till then, will the theoretical perfection and beauty, and the great practical merit, of Airy's correction of the compass by soft iron and permanent magnets (which theoretically assumes the length of the needle to be infinitely small in proportion to its distance from the nearest iron or steel) be universally recognized and have full justice done to it in practice.

* That is to say, terms consisting of coefficients multiplying the sines or cosines of six times and of eight times the ship's magnetic azimuth.

discovered in the Admiralty standard compass, " suggested the idea, that " the arrangement of the needles in that compass might produce, in the " case of deviations caused by a magnet or mass of soft iron in close " proximity to it, a compensation of the sextantal and octantal deviations; " and this, on the subject being mathematically investigated [on the " approximate hypothesis that the intensity of magnetization is uniform " through the length of each needle, and equal in the different needles], " proved to be the case, this particular arrangement of needles reducing " to zero the coefficients of the terms involving the square of the ratio " of the length of the needle to the distance of the disturbing iron; so " that this remarkable result was obtained, that the arrangement of " needles which produces the equality in the moments of inertia is, by a " happy coincidence, the same as that which prevents the sextantal devia- " tion in the case of correcting magnets, and the octantal deviation in " the case of soft iron correctors. The consequence is, that with the " Admiralty compass cards, or with cards with two needles each 30° " from the central line, correcting magnets and soft iron correctors " may be placed much nearer the compass than can safely be done with a " single-needle compass card, and that the large deviations found in iron " ships may be thus far more accurately corrected."

It will be understood that the preceding statement, even as an index of subjects, gives but a very incomplete idea of Smith's thirty years' work on magnetism. Further information is to be found in his papers in the Transactions and Proceedings of the Royal Society, some of them contributed in conjunction with Sabine or with Evans, others in his own name alone. In 1850 he published separately* an account of his theoretical and practical investigations on the correction of the deviations of a ship's compass, which was afterwards given as a supplement to the Admiralty " Practical Rules " in 1855. The large deviations found in iron-plated ships of war " having rendered necessary the use of the exact instead of " the approximate formulæ," this article was rewritten by Smith for the Compass Department of the Admiralty. It now forms Part III. of the ' Admiralty Manual for the deviations of the Compass,' edited by Evans and Smith, to which are added appendices containing a complete mathematical statement of the general theory, proofs of the practical formulæ, and constructions and practical methods of a more mathematical character than those given in the body of the work for ordinary use. A separate publication, of " Instructions for correcting the Deviation of the Compass," by Smith, was made by the Board of Trade in 1857.

It is satisfactory to find that the British Admiralty ' Compass Manual,' embodying as it does the result of so vast an amount of labour, guided by the highest mathematical ability and the most consummate

* Instructions for Computation of Tables of Deviations, by Archibald Smith. Published for the Hydrographic Office of the Admiralty.

practical skill, has been appreciated as a gift to the commonwealth of nations by other countries than our own. It is adopted by the United States Navy Department, and it has been translated into Russian, German, Portuguese, and French. Smith's mathematical work, and particularly his beautiful and ingenious geometrical constructions, have attracted great interest, and have called forth fresh investigation in the same direction, among the well-instructed and able mathematicians of the American, Russian, French, and German Navy Departments.

The laborious and persevering devotion to the compass problem, which has been shown by British mathematicians and practical men, by Sabine, Scoresby, Airy, Archibald Smith, by Captains Johnson and Evans of the Compass Department of the Admiralty, and by Townson and Rundell, who acted as secretaries to the Liverpool Compass Committee, has been an honour to the British nation in the eyes of the world. Referring particularly to the Liverpool Compass Committee, Lieut. Collet, of the French Navy, the French translator of the 'Admiralty Manual,' in a history of the subject which he prefixes to his translation, says :—" Aidé par des libéralités particulières, soutenu surtout par " cette sorte de ténacité passionnée, tout particulière à la nation anglaise, " qui, en inspirant les longues et patientes recherches conduit sûrement " au succès et sans laquelle tous les moyens d'action sont impuissants à " surmonter les obstacles, ce Comité fit paraître successivement trois rap- " ports qui fixèrent d'une manière définitive la plupart des questions con- " troversées, et qui indiquèrent nettement, pour celles qui restaient à " résoudre, la marche qu'il fallait suivre et les véritables inconnues du " problème." And in an official publication by the American Navy Department, containing an English translation of Poisson's memoir, followed by the whole series of papers, theoretical and practical, on ships' magnetism, which have appeared in this country, we find the following passage, which must be gratifying to all who feel British scientific work and appreciation of it by other nations, to be a proper subject for national pride :—" * * With the complex conditions thus introduced, and the " more exacting requirements of experience in their practical treatment, " came the necessity for constantly aiming at *that complete analysis of* " *the magnetic phenomena of the ship* which has been so prominent and " characteristic a feature of the English researches."

The constancy to the compass problem in which Smith persevered with a rare extreme of disinterestedness, from the time when Sabine first asked him to work out practical methods from Poisson's mathematical theory, until his health broke down two years before his death, was characteristic of the man. It was pervaded by that " ténacité passionnée " which the generous French appreciation, quoted above, describes as a peculiarity of the English nation ; but there was in it also a noble single-mindedness and a purity of unselfishness to be found in few men of any nation, but simply natural in Archibald Smith.

Honourable marks of appreciation reached him from various quarters, and gave him the more pleasure from being altogether unsought and unexpected. The Admiralty, in 1862, gave him a watch. In 1864 he received the honorary degree of LL.D. from the University of Glasgow. The Royal Society awarded to him the Royal Medal in the year 1865. The Emperor of Russia gave him, in 1866, a gold Compass emblazoned with the Imperial Arms and set with thirty-two diamonds, marking the thirty-two points. Six months before his death Her Majesty's Government requested his acceptance of a gift of £2000, as a mark of their appreciation of "the long and valuable services which he had gratuitously "rendered to the Naval Service in connexion with the magnetism of iron "ships, and the deviations of their Compasses." The official letter intimating this, dated Admiralty, July 1st, 1872, contains the following statement, communicated to Smith by command of the Lords of the Admiralty:—"To the zeal and ability with which for many years you "have applied yourself to this difficult and most important subject, My "Lords attribute in a great degree the accurate information they possess "in regard to the influence of magnetism, which has so far conduced to "the safe navigation of iron ships, not only of the Royal and Mercantile "Navies of this country, but of all nations."

The writer of this notice has obtained leave to quote also the following from a letter from the First Lord of the Admiralty, Mr. Goschen, of date February 23rd, 1872, announcing to Mr. Smith that the Government had determined to propose to Parliament that the sum of £2000 should be awarded to him "as a mark of recognition of the great and successful "labours" which he had "bestowed on several branches of scientific en-"quiry of deep importance to Her Majesty's Navy."

"I am aware that you have treated your arduous work in this direction "as a labour of love; and therefore I do not consider that the grant which "Parliament will be requested to sanction is in any way to be looked "upon as a remuneration of your services. . . . . I trust you will "regard it as a mark of recognition on the part of the country, of your "great devotion to enquiries of eminent utility to the public, conducted "in the leisure hours which remained to you in a laborious profession."

The following letter, which was addressed to the Editor of the 'Glasgow Herald,' and published in that paper last January, will be read with interest by others as well as those for whom it was originally written:—

"As an intimate friend of the late Archibald Smith of Jordanhill, I "desire to call your attention to a passage in your article of the "30th December upon him, which might perhaps convey a wrong im-"pression to the minds of your readers.

"You say that 'mathematics . . . in its application to practical navi-"'gation was the amusement of his lighter hours.' The truth is, that "the profession of a Chancery barrister, which the claims of a large "family forbade him to abandon, occupied his best energies from early

" morning till late in the evening—in other words, what would in the " case of most people, be called 'his whole time;' and compass investi- " gation of the most minute and severe nature, undertaken after mid- " night, and carried on far into the morning hours by a man whose brain " had been working all day, and must work again the next day, can " hardly be called 'the amusement of lighter hours.' The same remark " applies to vacations, during which his magnetic papers were constantly " with him—on railway journeys, on board the yacht, the last subject of " his thoughts at night, the first in the morning, at one time depriving " him, to an alarming extent, of the power of sleep; for, unlike the " labours of law, these abstruse subjects cannot be dismissed at will.

" The fact is that, in addition to the love of science for her own sake, " he was penetrated by the conviction of the usefulness of his work. " His splendid abilities, supported by a constitution of unusual vigour, " were freely and heartily devoted to the service of his country, and the " good of his fellow-creatures. 'Think how many lives it will save,' was " his answer to an anxious friend who begged him to relinquish labours " so exhausting, and to give himself ordinary rest. But the inevitable " result followed; and though in earlier days it had seemed as if nothing " could hurt his constitution, and his friends might anticipate for him " the length of days for which many of his family had been remarkable, " yet the continued mental strain did its work too surely, and in 1870 " he was compelled to give up his profession with shattered health, to " spend two short years with those he loved, and then sink into a prema- " ture grave. You observe that 'from the very commencement of his " career Her Majesty's Government (to their credit be it said) appreciated " the supreme importance of his researches.' In justice to the Govern- " ment, it ought also to be mentioned, that they asked [twelve years ago] " what acknowledgment should be made to him for work undertaken at " their request, and when Smith named *a watch*, it was presented to him by " the Admiralty. The testimonial presented to him during the past year, " 'not as representative of the money value of his researches, but as a " 'mark of their appreciation of their worth,' and still more, the graceful " letter in which Mr. Goschen intimated to him that it was awarded, " gave him pleasure, and his friends must always be glad that it did not " come too late.

" The truth is, Sir—and it is for this reason that I address you—that " services such as his, rendered at such heavy cost to himself and his " sorrowing friends, deserve the highest reward which can be given, " namely, the gratitude of the nation."

One more extract in conclusion. The following from the 'Solicitors' Journal and Reporter' of January 11th, 1873, contains a brief statement regarding the estimation in which Smith was held in relation to his legal profession, and concludes with words in which the writer of this article wishes to join, and therefore gives without quotation marks:—

" When Mr. James Parker was made Vice-Chancellor he appointed " Mr. Smith his Secretary; and he was also Secretary to the Decimal " Coinage Commission, which made its final report in 1859. In that " report there is a *résumé* of the subject by Mr. Smith; and one may see " there not only the special knowledge which he had collected on the " matter in hand, but an example of his thorough and exhaustive style, " close, compressed, and rich with fruits which it had cost him long " labours and careful thought to mature. Ungrudgingly and without " parade he used to offer the products of his toil: 'This,' he said to the " writer, pointing to one half page of figures in his book, 'cost me six " 'weeks of hard work.' It was thus he ever worked: no pains seemed " to be too much; and consequently a marvellous neatness and elegance " adorned all that he did. In his profession, although he did not attain " the same exceptional eminence as in science, there was much that " deserves notice. His mental characteristics were of course more or " less apparent here. As a draughtsman few could compare with him " for conciseness and perspicuity. His opinions were much esteemed; " and his arguments, though far from brilliant in manner, had in them " so much sound law and careful and subtle analysis that they were " always of interest and value, and commanded the respect and attention " of the judges. The important change which substituted figures for " words as to dates and sums occurring in bills in Chancery was made, " it is believed, at his suggestion. The well-known case of *Jenner* v. " *Morris* (on appeal 3 D. F. & J. 45, 9 W. R. 391), is an instance of one " of his successful arguments; and the case of *Deare* v. *Soutten* (9 L. R. " Eq. 151, 18 W. R. 203), in which the former case was reconsidered " and confirmed, illustrates the research and industry which he was wont " to use in all matters which came before him. A judgeship in Queens-" land was offered to him about the year 1864; but he declined it."

In private life those who knew Archibald Smith best loved him most; for behind a reserve which is perhaps incident to engrossing thought, especially when it is concerned with scientific subjects, he kept ever a warm and true heart; and the affectionate regrets of his friends testify to the guileless simplicity and sweetness of his disposition, which nothing could spoil or affect. About the close of 1870 he was compelled by illness to give up work; but two years later he had wonderfully rallied, and, building too much on a partial recovery of strength, had recurred imprudently to some of his old scientific pursuits. A few weeks before his death he revised the instructions for compass observations to be made on board the 'Challenger,' then about to sail on the great voyage of scientific investigation now in progress. The attack of illness which closed his life was unexpected and of but a few hours' duration. In 1853 he married a daughter of Vice-Chancellor Sir James Parker, then deceased; and he leaves six sons and two daughters. He died on the 26th of December, 1872.

Appendix.

1. *Smith's Deduction of Practical Formulæ from Poisson's Mathematical Theory.*

Let the components of the terrestrial magnetic force*, parallel to three rectangular lines of reference fixed with reference to the ship, be denoted by X, Y, Z; the components at the point occupied by the compass † of combined magnetic force of earth and ship by X′, Y′, Z′; the components of that part of the ship's action depending on "permanent" or "subpermanent" magnetism, by P, Q, R, quantities which mathematically must be regarded as slowly varying parameters, their variations to be determined for each ship by observation; and the components of that part of the ship's action which depends on transiently induced magnetism by p, q, r; so that we have

$$X'=X+p+P, \quad Y'=Y+q+Q, \quad Z'=Z+r+R. \quad . \quad . \quad . \quad . \quad (1)$$

Lastly, let (p, x), (q, x), (r, x) be the values which p, q, r would have if the earth's force were of unit intensity, and in the direction of x; (p, y), (q, y), (r, y) the same for y; and (p, z), (q, z), (r, z) the same for z. By the elementary law of superposition of magnetic inductions the actual value of p will be $(p, x)X+(p, y)Y+(p, z)Z$; and corresponding expressions will give q and r. Hence, and by (1), we have

$$\left.\begin{aligned} X' &= X + (p, x)X + (p, y)Y + (p, z)Z + P, \\ Y' &= Y + (q, x)X + (q, y)Y + (q, y)Z + Q, \\ Z' &= Z + (r, x)X + (r, y)Y + (r, z)Z + R. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

These equations were first given by Poisson in 1824, in the fifth volume of the Memoirs of the French Institute, p. 533. From these Smith worked out practical formulæ for the main case of application, that of a ship on even keel, thus: let

H be the earth's horizontal force;
H' the resultant of the earth's and ship's horizontal forces;
θ the dip;
ζ the ship's "magnetic course;"
ζ' the ship's "compass course;"
$\delta=\zeta-\zeta'$ the deviation of the compass.

* That is to say, the force experienced by a unit magnetic pole. The direction of the force is taken as that of the force experienced by a south pole, or (according to Gilbert's original nomenclature) the pole of a magnet which is repelled from the southern regions of the earth. British instrument-makers unhappily mark the north pole with S and the south with N.

† The length of the needle is supposed infinitely small in comparison with the distance of the nearest iron of the ship from the centre of the compass.

Then, if the directions of x be longitudinal from stern to head, y transverse to starboard, z vertically downwards, we have

$$X = H\cos\zeta,\quad Y = -H\sin\zeta,\quad Z=H\tan\theta,$$
$$X' = H'\cos\zeta',\quad Y' = -H'\sin\zeta'.$$

Resolving along and perpendicular to the direction of H we find, after some reductions,

$$\left.\begin{aligned}\frac{H'}{\lambda H}\sin\delta &= \mathfrak{A}+\mathfrak{B}\sin\zeta+\mathfrak{C}\cos\zeta+\mathfrak{D}\sin 2\zeta+\mathfrak{E}\cos 2\zeta,\\ \frac{H'}{\lambda H}\cos\delta &= 1+\mathfrak{B}\cos\zeta-\mathfrak{C}\sin\zeta+\mathfrak{D}\cos 2\zeta-\mathfrak{E}\sin 2\zeta,\end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

where

$$\left.\begin{aligned}\lambda &= 1+\frac{(p,x)+(q,y)}{2}, & \mathfrak{A} &= \frac{1}{\lambda}\frac{(q,x)-(p,y)}{2},\\ \mathfrak{B} &= \frac{1}{\lambda}\left[(p,z)\tan\theta+\frac{P}{H}\right], & \mathfrak{C} &= \frac{1}{\lambda}\left[(q,z)\tan\theta+\frac{Q}{H}\right],\\ \mathfrak{D} &= \frac{1}{\lambda}\frac{(p,x)-(q,y)}{2}, & \mathfrak{E} &= \frac{1}{\lambda}\frac{(q,x)+(p,y)}{2}.\end{aligned}\right\} \quad . \quad . \quad (4)$$

Dividing the first by the second, of (3) we find

$$\tan\delta = \frac{\mathfrak{A}+\mathfrak{B}\sin\zeta+\mathfrak{C}\cos\zeta+\mathfrak{D}\sin 2\zeta+\mathfrak{E}\cos 2\zeta}{1+\mathfrak{B}\cos\zeta-\mathfrak{C}\sin\zeta+\mathfrak{D}\cos 2\zeta-\mathfrak{E}\sin 2\zeta} \quad . \quad . \quad . \quad . \quad (5)$$

which gives the deviation on any given magnetic course, ζ, when the five coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are known. Multiplying both numbers by the denominator of the second member, and by $\cos\delta$, and reducing, we find

$$\sin\delta=\mathfrak{A}\cos\delta+\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'+\mathfrak{D}\sin(\zeta+\zeta')+\mathfrak{E}\cos(\zeta+\zeta), \quad . \quad (6)$$

or

$$\sin\delta=\mathfrak{A}\cos\delta+\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'+\mathfrak{D}\sin(2\zeta'+\delta)+\mathfrak{E}\cos(2\zeta'+\delta). \quad . \quad (7)$$

These give the deviations expressed nearly, though not wholly, in terms of the compass courses.

When the deviations are of moderate amount, say not exceeding 20°, equation (6) or (7) may be put under the comparatively simple and convenient form

$$\delta=A+B\sin\zeta'+C\cos\zeta'+D\sin 2\zeta'+E\cos 2\zeta', \quad . \quad . \quad . \quad . \quad (8)$$

in which the deviation is expressed wholly in terms of the compass courses; and this will be sufficiently exact for practical purposes.

It will be seen that the $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are nearly the natural sines of the angles A, B, C, D, E.

2. *Dygograms of Class* II.

Take lengths numerically equal to X, Y, Z and X′, Y′, Z′ for the coordinates of two points. The axes of coordinates being fixed relatively to the ship, conceive the ship to be turned into all positions round a fixed point taken as the origin of coordinates; or for simplicity imagine the ship to be fixed and the direction of the earth's resultant force to take all positions, its magnitude remaining constant: the point (X, Y, Z) will always lie on a spherical surface, [(9) below); and the point (X′, Y′, Z′) will always lie on an ellipsoid fixed relatively to the ship. For we have

$$X^2+Y^2+Z^2=I^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where I denotes the earth's resultant force. Now by (2) solved for X, Y, Z, we express these quantities as linear functions of

$$X'-P, \quad Y'-Q, \quad Z'-R.$$

Substituting these expressions for X, Y, Z, in (9) we obtain a homogeneous quadratic function of $X'-P$, $Y'-Q$, $Z'-R$, equated to I^2, which is the equation of an ellipsoid having P, Q, R, for the coordinates of its centre.

It is noteworthy that the point (X′, Y′, Z′) is the position into which the point (XYZ) of an elastic solid is brought by a translation (P, Q, R), compounded with a homogeneous strain and rotation represented by the matrix

$$\left(\begin{array}{ccc} 1+(p,x), & (p,y), & (p,z), \\ (q,x), & 1+(q,y), & (q,z), \\ (r,x), & (r,y), & 1+(r,z). \end{array}\right) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Instead of drawing at once the dygogram surface for the resultant of the force of earth and ship (X′, Y′, Z′), draw according to precisely the same rule, the dygogram surfaces for (X, Y, Z), the earth's force, and ($X'-X$, $Y'-Y$, $Z'-Z$), the force of the ship. The first of these will be a sphere of radius I. The second will be an ellipsoid having its centre at the point (P, Q, R). Let ON and OM be corresponding radius vectors of these two surfaces. On OM, ON describe a parallelogram MONK. OK is the resultant force of earth and ship at the point occupied by the ship's compass. Vary the construction by taking a "triangle of forces" instead of the parallelogram, thus:—Produce MO through O to m, making Om equal to MO; in other words, draw the dygogram surface representing ($X-X'$, $Y-Y'$, $Z-Z'$); and of it let Om be the radius vector corresponding to OM of the spherical-surface dygogram of the earth's force. Join Nm; through O draw OK equal and parallel to Nm. OK (the same line as before) is the radius vector of the resultant dygogram surface, corresponding to ON of the spherical dygogram. The law of correspondence between N on the spherical surface and m on the ellipsoid

is, according to (2) above, that m is the position to which M is brought—translation $(-P,\ -Q,\ -R)$ and strain* with rotation, represented by the matrix

$$\begin{pmatrix} (p,x), & (p,y), & (p,z), \\ (q,x), & (q,y), & (q,z), \\ (r,x), & (r,y), & (r,z). \end{pmatrix} \quad \ldots\ldots\ldots \quad (11)$$

Take any plane section (large or small circle) of the spherical surface. The corresponding line on the ellipsoid is also a plane section, but generally in a different plane from the other. For example, let the ship revolve round a vertical axis OZ; in other words, relatively to the ship let ON revolve round OZ in a cone whose semi- vertical angle is θ, the dip. The locus of N is a horizontal circle whose radius is H, the horizontal component of the earth's magnetic force. The corresponding locus of m is an ellipse, not generally in the plane perpendicular to OZ—that is to say, not generally horizontal. This ellipse and that circle are Smith's "Ellipse and Circle" (Admiralty Manual, 3rd edition, 1869, App. 2, page 168). The projection of the ellipse on the plane of the circle is the dygogram of what is wanted for the practical problem, namely the horizontal component of the ship's force.

By a curious and interesting construction (Admiralty Manual, page 175) Smith showed that, when $\mathfrak{A}$ and $\mathfrak{E}$ are zero, the ellipse and circle are susceptible of a remarkable modification, by which, instead of them, an altered circle and another circle (generally smaller) are found, with a perfectly simple law of corresponding points, to give, in accordance with the general rule stated above, the magnitude and direction of the resultant of horizontal force on the ship's compass. But in point of fact the comparison with Dygogram No. I., by which (pages 168, 169) Smith introduced Dygogram No. II., taken along with his previous mechanical construction of Dygogram No. I. (pages 166, 167), proves that Dygogram No. II., simplified to two circles, is not confined to cases in which $\mathfrak{A}$ and $\mathfrak{E}$ vanish, and so gives to this beautiful construction a greatly enhanced theoretical interest. It is to be also remarked that, although the necessity for supposing $\mathfrak{A}$ and $\mathfrak{E}$ zero has been hitherto of little practical moment, as their values are very small for ordinary positions of the compass in all or nearly all ships at present in existence, the greatly increased quantity of iron in the new turret ships, and its unsymmetrical disposition in the newest projected type (the 'Inflexible'), may be expected to give unprecedentedly great values to $\mathfrak{E}$ and $\mathfrak{A}$. The happy artifice by which Smith found two circles to serve for the "ellipse and circle" consisted in altering the radius of the first circle from H to λH. If, further, we alter it

* This strain must include reflexion in a plane mirror so as not to exclude negative values exceeding certain limits in the constituents of the matrix. It is to be borne in mind that, imaginary values of the elements being excluded, strain and reflexion can only alter spheres or ellipsoids to spheres or ellipsoids, not to hyperboloids.

in magnitude and direction, and make it represent the resultant of λH to north and $\mathfrak{A}$ to east, thus including part of the ship's force, namely $(\lambda-1)$H to north and $\mathfrak{A}$ to east, along with the earth's horizontal force in one circular dygogram, the residue of the horizontal component of the ship's force has also a circular dygogram. The construction thus obtained is fully described and illustrated by a diagram under the heading "Dygogram No. II., above. The proof of this is very simple. The following is the analytical problem of which it is the solution:—In the general equations (2) suppose Z to be constant, and put

$$X'-(p,z)Z-P=X'', \quad Y'-(q,z)Y-Q=Y''. \quad . \quad . \quad . \quad . \quad (12)$$

We have

$$\left.\begin{aligned} X'' &= [1+(p,x)]X+(p,y)Y, \\ Y'' &= \quad (q,x)X+[1+(q,y)]Y. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Now imagine two dygogram curves (ellipses or circles) to be constructed as the locus of points $(x, y,)$, (x', y') given by the equations

$$\left.\begin{aligned} &X^2+Y^2=H^2, \\ &x=X+(\alpha X+\beta Y); \qquad y=Y+(\gamma X+\delta Y); \\ &x'=X''-X-(\alpha X+\beta Y); \quad y'=Y''-Y-(\gamma X+\delta Y); \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

and let it be required to find α, β, γ, δ so that these two curves may be circles; we have four equations for these four unknown quantities. Then, as

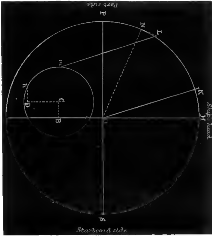
$$x'+x=X'', \quad y'+y=Y'',$$

the resultant of the radius vectors of the two concentric circles thus obtained is the resultant of the constituent (X'', Y'') of the force on the compass; and by (12) we have only to shift the centre of one of them to the point whose coordinates are $(p, z)\,Z+P$, $(q, z)\,Z+Q$, to find two circles such that the resultant of corresponding radius vectors through the centre of one of them shall be the whole horizontal component of the force on the compass. Thus we have Smith's beautiful and most useful Dygogram of two Circles.—W. T., January 1874.

$\lambda H = 1.$
$\mathfrak{A} = -0{\cdot}017.$
$\mathfrak{B} = -0{\cdot}408.$
$\mathfrak{C} = -0{\cdot}090.$
$\mathfrak{D} = +0{\cdot}156.$
$\mathfrak{E} = +0{\cdot}015.$

Rule for using Dygogram No. I.—In the diagram Q is a fixed point of the "Limaçon," called "the pole of the dygogram." It lies in the axis of symmetry, which is indicated by a dotted line. NQS, EQW are two lines through Q at right angles to one another; and O, P are two points, in positions fixed by the ship's magnetic elements. The length OP represents "mean force on compass to north" (λH). Take any point R on the curve, such that NOR is equal to the ship's "magnetic course;" then is ROP the "deviation" of the compass, and OR represents the horizontal component of the force on it.

Dygogram No. II.



Use.—Make $\widehat{hm}$ equal in angular value to $\widehat{NH}$; then OK, parallel and equal to mL, shows direction of needle and magnitude of horizontal component force on it when correct magnetic north is in direction ON, and ship's head OH: NOH being ship's "magnetic course," KOH is the corresponding "compass course."